Two Nucleons on a Lattice

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The connection between QCD and nuclear structure is one of the fundamental problems in theoretical physics. Recent progress in lattice QCD renews the hope of finally understanding nucleon-nucleon interactions on a firm theoretical basis. Besides the being a problem of principle, there are also more practical reasons to want first principle calculations with two or more nucleons. Some parameters used in nuclear modeling like exchange currents and three-body forces are difficult to be determined experimentally and any theoretical guide in this direction is welcome. Very little was made in this direction though. In this paper we took the first steps towards estimating what would take to learn about nucleon-nucleon interactions from lattice QCD.

The two-nucleon sector is near an infrared fixed point of QCD and as a result the S-wave scattering lengths are unnaturally large compared to the effective ranges and shape parameters. For this reason, it is usually assumed that a lattice QCD simulation of the two-nucleon sector will require a lattice that is much larger than the scattering lengths in order to extract quantitative information. Such expectation precludes the possibility of realistic simulations in the foreseeable future. In this paper we point out that this does not have to be the case: lattice QCD simulations on much smaller lattices will produce rigorous results for nuclear physics. The essential observation is that at distances larger than the pion Compton wavelength, QCD reduces to a very simple quantum mechanical problem, namely, the problem of two particles in a box of size L interacting through a (strong) short range potential. This problem is soluble and it is possible to related the energy levels of the two particles to the phase S-wave shifts at certain values of momenta. We provided the formula relating the energy levels to the phase shifts and analyzed it numerically. We found that indeed one can use lattice much smaller than the scattering lengths (about 5.4 fm and 20 fm in the spin singlet and triplet, respectively). By the other hand, in the spin singlet channel, lattice sizes of at least 7 fm would be needed to extract the scattering length from lattice QCD calculations, and even larger lattices if one is interested in phase shifts at finite values of momenta. The situation is similar but slightly worse in the spin triplet channel.

The estimated size of the lattices that seem to be required for the study nucleon-nucleon interaction from lattice QCD point in a very clear way to the need of cleverer methods and more extensive use of different effective theories that can help to make the connection between low-energy nuclear physics and QCD like, for instance, use of the "ε-regime", asymmetrical boxes, twisted boundary conditions, etc, all of which are being actively pursued.

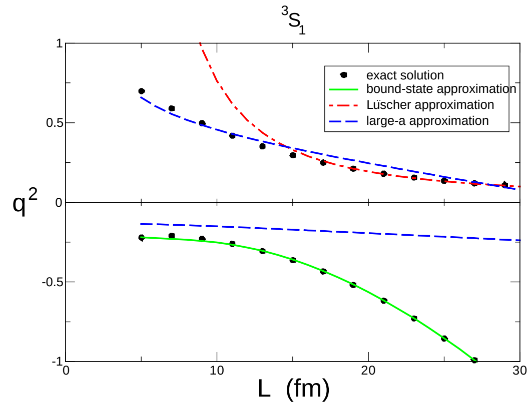

FIG. 1: The two lowest-lying energy eigenstates of two nucleons in the $^3S_1$ channel on a lattice of size L. The solid circles correspond to the exact energy levels assuming the experimentally measured scattering lengths and effective ranges. The other curves correspond to asymptotic approximations.

REFERENCES

[1] S. R. Beane et at., Phys.Lett. B585, 106-114 (2004).